# EFFECT OF FLEXIBILITY ON TRAJECTORIES OF MANIPULATORS

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## Abstract

In this study, a planar manipulator with two links is considered. The end point trajectory and the payload are defined as the inputs. The angular positions, velocities, and accelerations, and the joint forces and the necessary external motor torques are calculated by using the rigid body dynamics. The forces and torques acting on the links and the distributed inertial forces are used as the inputs to a finite element program to calculate the vibration of the links due to the flexibility. The effect of the vibration of the links on the end point trajectory is analysed. The deviation of the end point from the required trajectory is studied for different cross sectional areas, lengths of the links, and for different speeds.

## 1 Introduction

Flexibility effects increase as the weight of the robot manipulators decrease, loads increase or speeds increase. It is desired to design lighter robots carrying out heavier payloads at higher speeds. Thus, flexible link manipulators are a subject of intensive research. Designers of such manipulators need information about the dynamic response of the systems. Control actions can modulate the vibration effect of flexibility. There are many research works published on the dynamics and control of flexible manipulators. The review of the literature can be found in references [1-3]. Book et al. [4] linearized the equations of motion about a nominal configuration for a two- link flexible manipulator. Chang and Hamilton [1], and Usoro et al. [5] presented a finite element/ Lagrangian approach for the mathematical modelling for the manipulators with flexible links. Yigit [3] modelled a two- link rigid- flexible manipulator and derived the equations of motion by applying the Hamilton's principle. Xi and Fenton [6] studied the inverse kinematics of a Puma-type flexible link manipulator under large link deflections. Huston and Wang [7] summarised procedures for studying flexible multibody systems using finite segment modelling. In these procedures flexible members are themselves modelled as multibody lumped systems. The papers using different iterative methods for flexible manipulator dynamics are introduced in Ref. [2]. Gawronski et al. [2] derived a linear time- varying model of a flexible two-link manipulator.

In this study, a two-link flexible manipulator is considered. The dynamics of the manipulator is studied by the computer aided design (CAD) procedure. First, a database file is created for the end point trajectory, velocity and acceleration, and the payload. Then, a database file is constructed to define the geometry and material properties of the manipulator, and the external forces and torques. A general purpose dynamic analysis computer program is used to find the angular positions, velocities, and accelerations of the links, and the joint forces, the necessary external torques, and the distributed inertial forces. The rigid body dynamics is used in this program. The time varying (periodic) forces, torques and distributed inertial forces acting on the links are used as the input information for an available finite element (FE) program (ANSYS 5.0) [8] to study the vibration of the links. The output data from the FE program is used to analyse the effect of flexibility on the end point trajectories.

## 2 Computer Aided Dynamic Analysis

The sketch of the manipulator considered is shown in Fig.1. The element numbers for the links are 2 and 3, respectively. $\theta_i$ is the angular position of the i-th element, and $T_i$ is the external torque acting on the i-th element. $\mathbf{R}_B$ is the position vector of the end point, and $\mathbf{F}_B$ is the payload. $l_i$ is the length of the link i; thus, $l_2$=OA and $l_3$=AB. $m_i$ is the mass and $I_i$ is the moment of inertia of the element i.

First, the input file is created to define the position, velocity, and acceleration of the end point, and the payload. The structure of the file is given in File 1 named as M2L.INP below. The values are given as an example.

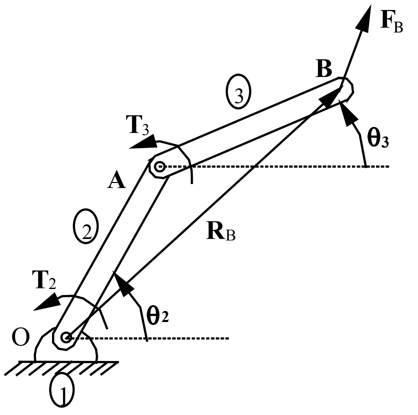

**Figure 1 The sketch of the manipulator**

**File 1: M2L.INP**

```
32, 4
1, T=0
3, 0.496, 0, -394.7843
3, 0.159, 6.2832, 0
1, 0
1, -50
2, T=0.003125
...
...
```

In File 1, the numbers 32 and 4 in the first line indicate that the calculations are done for 32 steps, and 4 sets of inputs exist. For the manipulator considered 2 kinematic inputs (x and y coordinates of the end point), and 2 kinetic (force/torque) inputs (x and y components of the payload) exist. In the second line, 1 indicates the step number, and T=0 gives the value of the time for this step. The following 4 lines give the kinematic and kinetic input values for this step. The lines starting with 3 indicate kinematic inputs (position, velocity, and acceleration information), and the lines starting with 1 indicate force/torque inputs. For the first step, the x coordinate of the end point is 0.496 m, the velocity and acceleration in the x direction are 0 m/s and -394.7843 m/s$^2$, respectively. The y coordinate is 0.059 m, the velocity and acceleration in the y direction is 6.2832 m/s and 0 m/s$^2$. The x and y components of the payload are 0 and -50 N, respectively, for the first step. The value of time for the second step is 0.0032258 s. The varying values of the kinematic and force/torque inputs follow in the same format as in the first step.

After preparing File 1, the file defining the manipulator system is created. File 2 named M2L.SYS is given below for this purpose. The information M2L.INP in the first line indicates the input file name above, and M2L.OUT is the output file name where the results of the dynamic analysis program are stored. There are 3 elements in the system. Element 1 is always the frame. Elements 2 and 3 are the links as numbered in Fig. 1. In File 2, the length, the distance of the center of mass from the link origin, the mass, and the moment of inertia are given as 0.3 m, 0.15 m, 1.5 kg, and 0.01125 kg-m$^2$ for the element 2 (Link 2), respectively. The values for the link 3 are given in the similar format.

**File 2 : M2K.SYS**

```
M2L.INP, M2L.OUT
'---------------------
3 ELEMENTS
EL. 1, FRAME
EL. 2, LINK, 0.3, 0.15, 1.5, 0.01125
EL. 3, LINK, 0.4, 0.20, 2.0, 0.02667
'-----------------------
0 SL VALUES
'-----------------------
0 FI  VALUES
'-----------------------
2 KINEMATIC INPUTS
RBX,RBY
'-----------------------
1 VECTOR POLIGON
3 VECTORS
+, P, LL2, TH2
+, P, LL3, TH3
-, C, RBX, RBY
'--------NITER1,NITER2/ XB(),XH()
5, 20
1.047, 0.001
5.75,  0.001
'-- POSITIONS OF CENTER OF MASSES
EL. 2, 1 VECTOR
+, P, LG2, TH2
EL. 3, 2 VECTORS
+, P, LL2, TH2
+, P, LG3, TH3
'----------------------
2 JOINTS
1, 2, RJ, 0, 0
2, 3, RJ, LL2, 0
'-----------------------
2 EXTERNAL TORQUES
EL. 2, T2,  0
EL. 3, T3, 0
'-----------------------
2 EXTERNAL FORCES
EL. 3, FBX,  0, LL3
EL. 3, FBY, 1.57, LL3
'-----------------------
2  FORCE/ TORQUE INPUTS
FBX,FBY
```

There are no constant distances (SL values) and constant angles (FI values) in the system. Two kinematic inputs, RBX and RBY, which correspond to the x and y coordinates of the end point ($x_B$ and $y_B$ , respectively) are defined.

From Fig.1, the following equation can be written:

$$\mathbf{R}_{\mathrm{A}} + \mathbf{R}_{\mathrm{BA}} - \mathbf{R}_{\mathrm{B}} = \mathbf{0} \tag{1}$$

where $\mathbf{R}_{\mathbf{A}}$ is the position vector of A with respect to O, $\mathbf{R}_{\mathrm{BA}}$ is the position vector of B with respect to A, and $\mathbf{R}_{\mathrm{B}}$ is the position vector of B with respect to O. Eq. 1 can be written with complex numbers [9] as:

$$l_2 e^{i\theta_2} + l_3 e^{i\theta_3} - \left(x_B + i y_B\right) = 0 \tag{2}$$

where i $= \sqrt{-1}$ . Eq. 2 is defined in File 2 after the line written as "1 VECTOR POLYGON". In the following lines P indicates polar coordinates, C indicates cartesian coordinates. For example LL2 corresponds to $l_2$ (the length of the link 2) and TH3 corresponds to $\theta_3$ (the angular position of the link 3). The real and imaginary parts of Eq. 2 give 2 equations to solve $\theta_2$ and $\theta_3$. The equations are nonlinear and can be solved by the Newton-Raphson method [10]. The minimum and maximum number of iteration numbers are defined as 5 and 20 in File 2. The initial guesses for $\theta_2$ and $\theta_3$ are 1.047 and 5.75 radians, respectively, and the acceptable error values are 0.001 rad. The derivative of Eq. (2) with respect to time is called the velocity equation and it is used to solve $\omega_2 = d\theta_2/dt$ and $\omega_3 = d\theta_3/dt$. The second derivative of Eq. (2) with respect to time is called the acceleration equation and it is used to solve $\alpha_2 = d\omega_2/dt$ and $\alpha_3 = d\omega_3 / dt$. The velocity equation is linearly dependent on $\omega_2$ and $\omega_3$, and the accelaration equation is linearly dependent on $\alpha_2$ and $\alpha_3$. In File 2, the position vectors of the center of masses are defined in the similar way as for the Eq. 2. Here, for example, LG3 is the distance of the center of mass of the link 3 from its origin.

There are 2 joints in the system under study, which are defined in File 2. For example, the elements 2 and 3 are joined by a revolute joint (RJ), and the location of the joint in Link 2 is at the distance $l_2$ (LL2) from its origin, and in the link 3 is at its origin. There are 2 external torques. The variables $T_2$ and $T_3$ correspond to the torques acting on the elements 2 and 3, respectively. 1 external force (the payload) exist, but it is defined by 2 external forces in File 2. FBX is the x component, and FBY is the y component of the payload. It is acting on the link 3 at the distance $l_3$ (LL3). The direction angle of these components with respect to the x axis are 0 and 1.57 radians, respectively. The force/torque inputs are also indicated in File 2.

A general purpose computer program is used to find the time-varying positions, velocities, and accelerations, and the external torques and the joint forces from the information given in Files 1 and 2. The program was developed by the first author. It can be used for simple mechanical systems in planar motion. The output file of the program gives the values of the angular positions, angular velocities, angular accelerations, acceleration vector components of the center of masses, external torques, and the longitudinal and vertical components of the joint forces for each time considered in the steps. The program uses equations of motions applied to the free body diagrams of the elements. The resulting equations are linearly dependent on the unknown forces and torques.

For each link, the geometry and material information, the external torque and joint forces acting on the link, and the distributed inertia forces are defined to the finite element (FE) program (ANSYS 5.0) using its procedures [9]. The FE program gives the time varying displacements, velocities, accelerations, and strains at the nodes assigned.

The required trajectory of the end point is called the rigid body (RB) trajectory. The displacement of the end point calculated by the FE program is added to the RB-trajectory to obtain the flexible body (FB) trajectory.

# 3 Results and Discussions

For obtaining the following results, it is assumed that the links are homogeneous and have constant cross sectional areas. The properties of the material of the links are assumed to be as $E = 200 \times 10^9$ N/m$^2$, $\rho = 7800$ kg/m$^3$, $\nu = 0.35$; where E is the elasticity modulus, $\rho$ is the density, and $\nu$ is the Poisson's ratio. 8 nodes are assigned in the FE program.The end point trajectory is

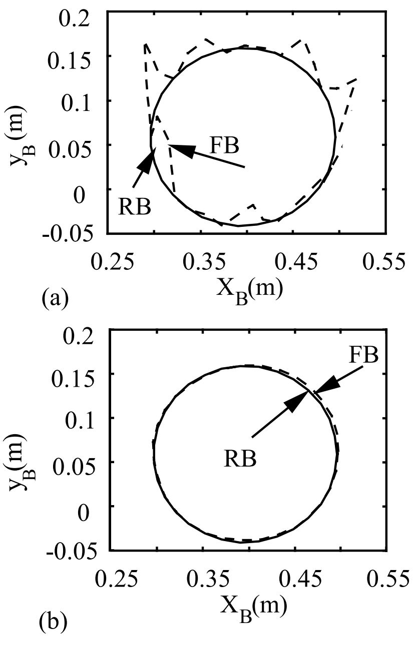

**Figure 2 RB and FB- trajectories for a) A = 500 mm$^2$ and b) A= 1050 mm$^2$**

assumed to be a circular path for Figures 2 and 3, and a linear path for Fig. 4.

The motion is taken as periodic. 32 points are considered in the period. The cycle speed is 600 cycles per minute, and $l_2 = 0.3$ m, $l_3 = 0.4$ m, unless it is stated. The displacement amounts due to the vibration are multiplied by 100 and added to the RB-trajectory in the figures in order to observe the effect of vibration clearly.

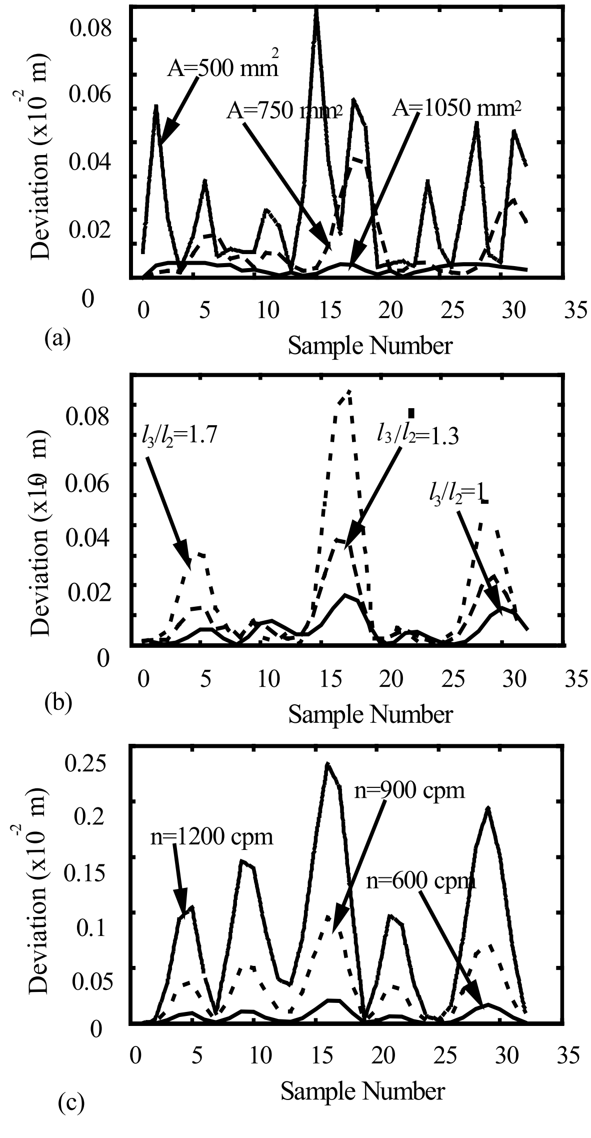

**Figure 3 Effect of a) cross sectional area, b) ratio of link lenghts, and c) speed of cycle on deviations for circular trajectory**

For the circular trajectory, $x_B = 0.396 + 0.1\cos\left(2\pi nt/60\right)$, and $y_B = 0.059 + 0.1\sin\left(2\pi nt/60\right)$ meters. $n$ is the value of cycles per minute (cpm) and $t$ is the time. The payload is taken as perpendicular to the trajectory, with a constant magnitude of 50 N. The force is assumed to be in the direction giving a clockwise moment with respect to the center of the circular trajectory. The RB and FB-trajectories are shown in Fig. 2 for different cross sectional areas (A). The deviations defined as the absolute values of the differences between RB and FB-trajectories in the radial direction, are studied in Fig. 3 for different cases. 32 sample points are considered

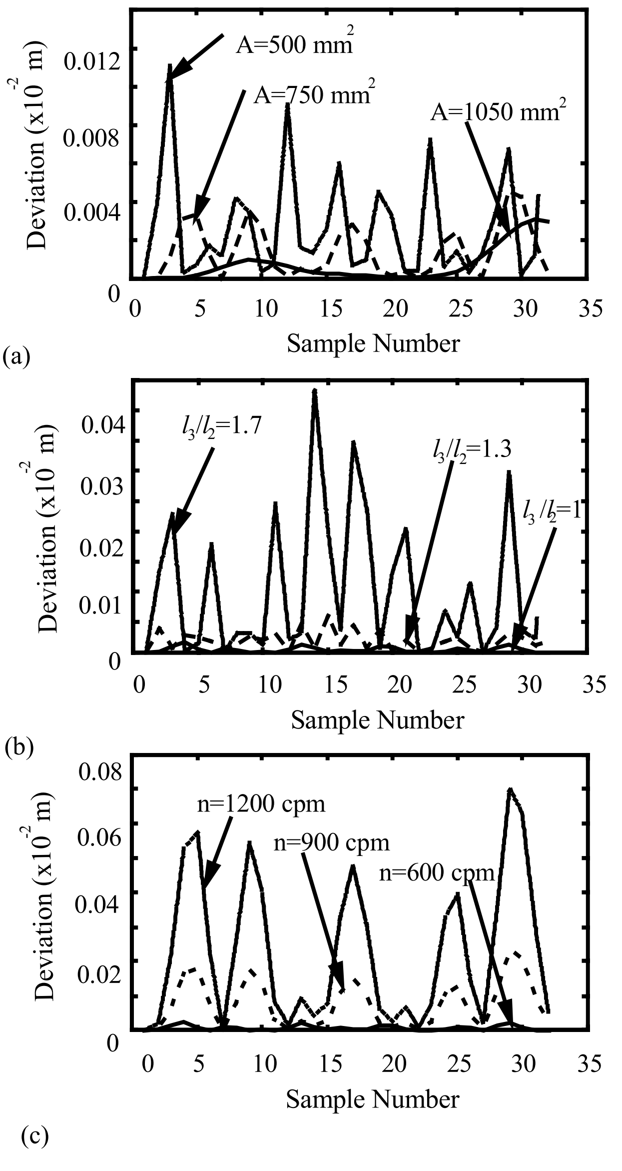

**Figure 4 Effect of a) cross sectional area, b) ratio of link lenghts, and c) speed of cycle on deviations for linear trajectory**

As observed from Fig. 3, the deviations increase as the cross sectional areas decrease, the $l_3/l_2$ ratio increase, or the speed of the cycle of the motion increase. These results are expected, because decreasing cross sectional areas mean increasing flexibility and thus larger deflections. Increasing speeds results in higher forces and thus larger deflections.

Similar results are obtained for different trajectories. The end point trajectory is taken as a line in the vertical direction (y direction) for the results shown in Fig. 4, where $\mathbf{x}_B$ = 0.412 m, and $\mathbf{y}_B$ = 0.21 - 0.1 cos (2πnt/60) m. A force (payload) with a magnitude of 50 N in the negative direction is taken while the end point is moving upwards, and the force is zero while the motion is downward. The deviations from the RB- trajectory is defined here as the absolute values of the differences between the RB and FB trajectories in the horizontal direction.

## 4 Conclusions

Computer aided design (CAD) techniques are developing very rapidly. Solid modelling, assembly, dynamic analysis, and finite element programs can be successfully used to study the effects of the flexibility in manipulators. In solid modelling and assembly programs input files for the dynamic analysis (Files 1 and 2 given in Section 2) can be created by computer graphics input techniques.

CAD programs can be used for the optimal design of manipulators. Condition monitoring by vibration measurement strategies may be developed. Integration of control actions to computer aided mechanical design programs may be done.

## References

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